WNE Linear Algebra
Final Exam
Series B

1 February 2022

Problems

Please use separate files for different problems. Please provide the following data in each pdf file

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Each problem is worth 10 marks.

Problem 1.
Let $V = \text{lin}((1,1,4),(1,2,-1),(2,3,3),(3,6,-3))$ be a subspace of $\mathbb{R}^3$.

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) find a system of linear equations which set of solutions is equal to V.

Problem 2.
Let $V \subset \mathbb{R}^4$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} 2x_1 + 3x_2 + 5x_3 + 2x_4 = 0 \\ x_1 + 2x_2 + 2x_3 + x_4 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) for which $t \in \mathbb{R}$ does the vector $v = (-5, t^2, 1, 1)$ belong to V? For all such t find the coordinates of vector v relative to the basis $\mathcal{A}$.

Problem 3.
Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear endomorphism given by the formula

$$\varphi((x_1,x_2)) = (4x_1 + 3x_2, -6x_1 - 5x_2).$$

Let

$$A = M(\varphi)_{st}^{st}.$$

a) find the eigenvalues of φ and bases of the corresponding eigenspaces. Is matrix A diagonalizable?
b) compute A^{60}, where $A = M(\varphi)_{st}^{st}$.

Problem 4.
Let $\mathcal{A} = ((1,1),(1,0)), \mathcal{B} = ((1,3),(1,4))$ be ordered bases of $\mathbb{R}^2$. Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^3$, $\psi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be linear transformations given by the matrices

$$M(\varphi)_{\mathcal{A}}^{st} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad M(\psi)_{st}^{\mathcal{B}} = \begin{bmatrix} -1 & -1 \\ 1 & 2 \end{bmatrix}.$$

a) find the formula of φ,
b) find the matrix $M(\varphi \circ \psi)_{st}^{st}$.

Problem 5.
Let $V = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid -x_1 + 2x_2 + 3x_3 = 0\}$ be a subspace of $\mathbb{R}^3$.

a) find an orthonormal basis of V,
b) compute the orthogonal projection of $w = (0, 7, 0)$ onto V.

Problem 6.
Consider the following linear programming problem $2x_1 + x_2 \to \max$

$$\begin{cases} x_1 - x_2 \leqslant 4 \\ x_1 + 3x_2 \leqslant 12 \end{cases} \text{ and } x_1, x_2 \geqslant 0.$$

a) find the standard form of the problem and sketch the feasible region. Find a basic feasible set of the problem in the standard form.
b) solve the linear programming problem using **simplex method**. Which vertex of the feasible region corresponds to the optimal solution?